# REVIEWS

**An Introduction to Fluid Dynamics.** By G. K. BATCHELOR. Cambridge University Press, 1967. 615 pp. 75*s.* or $13.50.

The publication of Professor Batchelor's book is clearly an outstanding event. Even though it is addressed to students in applied mathematics it will have a much wider impact. Anyone concerned with fluid dynamics in any way and at any level will find much useful and interesting information here. I do not know of anywhere else, for example, where one can find an integrated discussion of bubble flows or a coherent up-to-date account of the flow past bodies over the whole range of Reynolds numbers. But, in accordance with the aims of the book, perhaps it should be considered primarily from the point of view of students and from the point of view of applied mathematics.

The book is essentially in two parts. The first three chapters lead from the physical concepts to the mathematical formulation of fluid motion. This is done in detail from a general point of view because it is designed to provide a firm basis for the later development of any of the main branches of fluid dynamics. These chapters culminate in the Navier–Stokes equations for a compressible fluid as the simplest comprehensive set to cover the major phenomena in fluid motion. But, the reader is kept aware of their limitations, and there is a good interplay with the more elementary aspects of the kinetic theory of gases to provide the valuable insight into the nature of the stress tensor and heat flux, and to show the roles of viscosity and heat conduction in the transfer of molecular momentum and energy. On the whole, this is familiar material but the presentation is more complete than usual. Bulk viscosity is examined, not glossed over; surface tension is explained clearly, and so on. There is a long section (50 pages) on the purely kinetic deductions of the velocity distribution from a given distribution of dilatation and vorticity, which, for me, interrupted the smooth development of the transfer of physical ideas into mathematical terms. I thought that, with this amount of detail, the section would have been better placed later, but it is perhaps a matter of taste.

These three chapters, then, give the fundamentals and are available for general use in any of the special branches of the subject. The remainder of the book is devoted to one such branch: incompressible flow. The idea is that parallel developments of other branches could spring from the same basic material, and some of these may be provided by the author subsequently. The major theme of this second part is the interaction of viscous and inertia forces. It contains a study of all possible applications involving flows in tubes or around bodies, as well as a remarkable variety of more exotic applications where similar ideas may be used with success. A final chapter introduces those areas where the combination of vorticity and inertia is the basic phenomenon. In these sections, the novelty of the applications and the fresh information on traditional topics makes a lively account of what could have appeared to be old-fashioned and 'academic' in the worst sense. A glance at the excellent plates is a sufficient

indication of the mixture of traditional and new, ranging from Prandtl's original photographs of vortex flow behind bodies and airfoils, through the mushroom clouds of vortex rings, to vortex breakdown on delta wings.

Perhaps it is worth noting some of the less usual topics to give the flavour. Along with the discussions of the viscous properties of ordinary fluids, there are accounts of Einstein's formula for the effective viscosity of a fluid with suspended particles and a derivation of the effective expansion viscosity for liquids containing small bubbles. There is also a non-mystical account of Darcy's law for porous media. Among the boundary-layer phenomena, the Ekman layer, oscillating layers, acoustic streaming, layers at free surfaces and on the boundaries of bubbles are all included. Many aspects of bubble flow were new to me. I had no idea that the drag on a bubble could be determined from the dissipation in the surrounding fluid as calculated from the *irrotational* flow, or that large bubbles adopted the spherical cap shape and that simple potential flow arguments could be used to obtain the rate of rise. This type of application certainly gives life to the tired old topic of flow round a sphere. In the discussion of momentum balance arguments, shaped charges combine nicely with water sprinklers. Cavitation and explosion bubbles provide some unsteady potential flow problems. Geophysical problems in rotating fluids introduce topics like Rossby waves to add to the more familiar study of vortex motion. I think the availability in one place of this broad range of applications, which are both important and illustrative, will make the book much more than a text for students.

I see no point to listing all the more usual topics, even though naturally they make up most of the book. They are all here and they are all well done. The lessons to be learnt from the works by Prandtl, Lamb and Goldstein are combined, developed and brought up to date. Readers of this *Journal* do not need to be assured of the editor's personal point of view, or of his aversion to scissors and paste!

The material is motivated and ordered with a view to the importance for fluid flow, not with an eye on the mathematical methods. For example, exact solutions to the Navier–Stokes equations are included when they arise in the particular flow problems being discussed, rather than placed in a special chapter as museum pieces. Again, viscous flow is studied first because it is essential to the understanding of inviscid results, even though the mathematics is harder and less precise. I think that these are certainly the correct choices. It does mean as a consequence, however, that more effort has to be spent on displaying the structure of the mathematical methods employed. This is an important question in a book addressed specifically to applied mathematicians. Fluid dynamics has a long and colourful history as the source of important mathematical ideas and techniques, and this is one reason for its widespread study. I do not think this aspect appears strongly enough here. In this respect, I have the feeling that the student is shown what other people have done, but is not shown how to develop an approach to solving problems or discovering new methods. The curse of 'dry water' has to be avoided at all costs, but I think the mathematical themes could have been exploited further without danger. This

comment is strictly with a view to the needs of students of applied mathematics; as noted above, this book will have a much broader audience.

The presentation is painstaking and thorough with every effort made to make the information complete and the arguments unambiguous. These efforts are entirely successful, and I think that the level of care and patience is the distinguishing characteristic of the book. Perhaps as a consequence, it is also very long. It runs to 600 pages and one must bear in mind that the extensive areas of acoustics, compressible flow, shock waves, water waves, hydrodynamic stability and turbulence are yet to come, even if we leave aside topics like magneto-hydrodynamics or flows with chemical reactions which require a body of knowledge in other fields. For this reason, I think many students will find its value more as a reference for large sections of material than as a first introduction. Some will not yet be committed to such a degree of immersion in the subject. Others, particularly in applied mathematics, will be concerned with the development of similar mathematical ideas in different fields. For these, a briefer selection of key ideas from all branches of the subject will be essential. However, in reviews one should not quibble as to how a book will be used, if one is convinced that it will be widely used and that the work is of high quality. I have no doubts in these respects.

G. B. WHITHAM

## Laminar Boundary Layer Theory. By H. L. EVANS. Addison-Wesley, 1968. 229 pp. £5. 17s.

The intention of the author of this book is to provide senior undergraduate and graduate engineering students with an understanding of the foundations of boundary layer theory. Of the nine chapters, the first is devoted to motivation and the second to a derivation of the boundary layer momentum and energy equations for small temperature differences and negligible dissipation. The next six are concerned with similar solutions of the equations for both two-dimensional and axisymmetric flow, and in the final chapter an approximate method is presented for solving the equations for general flows.

This brief description indicates that the author gives a somewhat unbalanced account of boundary layer theory, a criticism which might be muted if the various topics received satisfactory treatment. However, a more detailed investigation into the contents of the book reveals that unfortunately this is not so. In the first chapter study of boundary layer theory is motivated by an unconvincing qualititative argument, and certain assumptions (not all of which are independent) are made, from which the boundary layer equations are constructed in an *ad hoc* manner in chapter 2. The Reynolds number is mentioned frequently in these introductory chapters though not defined there, nor is there any discussion of the significance which this parameter may have. In the final formulation of the equations the boundary conditions are incomplete.

The author justifies his preoccupation with similarity solutions, a topic to which more than half of the book is devoted, on the grounds that such solutions afford insight into the behaviour of boundary layers in general and are used as

a basis for approximate methods. The importance of the latter is alleged to stem from the fact that to solve the boundary layer equations for a general two-dimensional flow is a major computational task, 'Indeed the amount of work involved is so great that it is *rarely attempted*' (my italics). The asymptotic role of similarity solutions receives no attention. The most disturbing features of these chapters are associated with the numerical solution of the Falkner–Skan equation and the corresponding energy equation. Numerical solutions are presented for a range of negative values of the Falkner–Skan parameter $\beta$ where it is, in fact, generally accepted (unless algebraic decay is suppressed) that the solution is not unique and the author's statement that no solutions of the energy equation exist for this range is plainly incorrect.

In the final chapter there is an extensive discussion of an approximate method due to Merk (*J. Fluid Mech.* vol. 5, 1959, p. 460) for handling non-similar two-dimensional boundary layer flows. This method is claimed by the present author to be one of the most accurate methods available and to be more economical than, say, the series solutions proposed by Blasius and Görtler (notwithstanding this remark, results from a three-term Blasius series are used as a reliable yardstick for comparison). The most unsatisfactory aspect of this chapter is the lack of any discussion of separation. In the first chapter it was noted that one of the limitations of an inviscid theory was its inability to predict separation and the purpose of studying boundary layer theory was to overcome such limitations. The reader who seeks information about the nature of the flow in the neighbourhood of the separation point will search in vain. Apart from a brief mention of a special case on page 178 there is not even a suggestion that in general it may be expected that the wall shear stress vanishes at the separation point. Much of the chapter is devoted to the estimation of such quantities as viscous drag, heat and mass transfer in regions away from separation. A recurring theme is that the method under discussion will not yield accurate results as separation is approached.

The reviewer's opinion is that the book adds little of value to the existing literature and will not be sought in preference to the more satisfactory, comprehensive and more modestly priced books already available.

N. RILEY